B.Sc Part-I PAPER-II

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~~Notes I~~

General Properties of definite integrals

Some General properties of definite integrals are Given below

Th.1 The value of a definite integral depends only on the limits and not on the variable of integration That is,

$$\int_a^b f(x)\,dx = \int_a^b f(y)\,dy$$

Let $\int f(x)\,dx) = F(x) + C$

then $\int_a^b f(x)\,dx = \Big[F(x) + C\Big]_a^b = F(b) - F(a)$

Again as $\int f(x)\,dx = F(x) + C$

therefore $\int f(y)\,dy = F(y) + C$

$$\therefore \int_a^b f(y)\,dy = \Big[F(y) + C\Big]_a^b = F(b) - F(a).$$

Hence the theorem.

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Th. 2 The interchanging of the limits changes the sign of the definite integrals

That is $$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx.$$

Let $\int f(x)\,dx = F(x) + C$

$$\therefore \int_a^b f(x)\,dx = \Big[F(x)+C\Big]_a^b = F(b)-F(a)$$

and $$\int_a^b f(x)\,dx = \Big[F(x)+C\Big]_b^a$$

$$= F(a) - F(b)$$

$$= -\Big[F(b)-F(a)\Big]$$

$$= -\int_a^b f(x)\,dx.$$

Thus the theorem.

Th. 3 $\int_a^b f(x)\,dx = \int_a^{c_1} f(x)\,dx + \int_{c_1}^b f(x)\,dx$

where $a \le c_1 \le b$.

Let $\int f(x)\,dx = F(x) + C$

then $\int_a^b f(x)\,dx = \Big[F(x) + C\Big]_a^b$

$= F(b) - F(a)$

and $\int_a^{c_1} f(x)\,dx + \int_{c_1}^b f(x)\,dx$.

$= \Big[F(x) + C\Big]_a^{c_1} + \Big[F(x) + C\Big]_{c_1}^b$

This prove the theorem.